ON THE CALCULATION OF LOW-ENERGY NEUTRON SCATTERING BY A COMPLEX POTENTIAL

ARUNDHATI GHOSH AND S. K. DUTTA

DEPARTMENT OF THEORETICAL PHYSICS,
INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE
JADAVPUR, CALCUTTA-32

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ABSTRACT. The scattering of low-energy neutrons by a potential $V(r) = -(V+iW)[1+e^{(r-R)/a}]^{-1}$ is calculated by applying a method of Lanczos (1938) for solving the Schrödinger equation. The neutron strength function $\overline{\Gamma}_n{}^0/D$ which is the ratio of the average value of neutron width to level spacing is obtained from the scattering amplitude. That this analytic method of solution is fairly exact is borne out by the fact that our results agree closely with the finding of Feshbach, Porter and Weisskopf (1954) who have solved numerically the differential equation with the same potential.

INTRODUCTION

A number of investigations (Feshbach *et al.*, 1954; Feshbach, 1958) have been made in interpreting the interaction of slow neutrons with a nucleus taking the nuclear potential to be complex. The resonance structure of neutron strength function has been first discussed by Feshbach, Porter and Weisskopf (1954) and they calculated the strength function with a complex square well potential. Later on Feshbach (1958) have solved numerically the Schrödinger equation with the Woods-Saxon potential and the results so obtained have nearly the same general pattern as those of the complex square well. The agreement with experimental findings is fair, Feshbach obtained for the value of $\overline{\Gamma}_n{}^\circ/D$ at $A = 155$,one maximum whereas the experimental results indicate two small peaks.

The object of the present paper is to solve analytically the Schrödinger equation with the Woods-Saxon potential for the case of positive energy states, the method employed here is due to Lanczos (1938) which we have already applied to solve the bound state problem (1960). Previously Lawson (1956) has obtained a solution of this problem in the form of infinite series the terms of which converge very badly, as such the solution of Lawson is not so useful in its application to physical problems. In our case if we take the solution up to eighth term the error in the differential equation is of the order of 1 in 10^6. Our results agree well with similar ones of Feshbach (1958).

In the theory of Feshbach, Porter and Weisskopf (1954) it is shown that for very low energy neutrons the scattering amplitude averaged over resonances is given by

$$\bar{\eta}_0 = e^{-2iKR'}(1-\pi\bar{\Gamma}_n/D) \quad \ldots \quad (1)$$

This gives $\bar{\Gamma}_n/D$ which is the ratio of the average value of neutron width to level spacing and R'/R where R' has the dimension of length and is a slowly varying function of energy. The magnitude of R' is of the order of nuclear dimension and it plays the role of scattering length; K is $\left(\frac{2M}{\hbar^2}E\right)^{\frac{1}{2}}$. The average total cross section (Feshbach *et al.*, 1954) is

$$\sigma_t^{(0)} = \frac{\pi}{K^2}\{|1-\bar{\eta}_0|^2+1-|\bar{\eta}_0|^2\}$$

$$= 4\pi R'^2+\frac{2\pi^2}{K^2}\bar{\Gamma}_n/D \quad \ldots \quad (2)$$

MATHEMATICAL FORMULATION AND RESULTS

The interaction potential between the neutron and the nucleus is taken as

$$V(r) = -(V+iW)[1+e^{(r-R)/a}]^{-1}$$

where R is a measure of the nuclear size and a is the diffusivity parameter. The Schrödinger equation for $l = 0$, scattering with the above potential may be written as

$$\frac{d^2u}{dx^2}+\frac{\lambda^2}{1+\beta e^x}u+K'^2u = 0 \quad \ldots \quad (3)$$

where $x = r/a$; $K'^2 = \frac{2M}{\hbar^2}Ea^2$; $\lambda^2 = \frac{2M}{\hbar^2}(V+iW)a^2$; $\beta = e^{-R/a} = e^{-x_0}$

The wave function $u=r\psi$ satisfies the necessary boundary conditions for scattering at $x=0$ and $x\rightarrow\infty$.

In the region $R \leqslant r < \infty$ we may write the solution as

$$u^0(x) = e^{-iK'x}F_- - \bar{\eta}_0 e^{iK'x}F_+ \quad \ldots \quad (4)$$

where $\bar{\eta}_0$ is the scattering amplitude averaged over neutron resonance energies.

We now make a transformation of the independent variable from x to $p = e^{-(x-x_0)}$ such that the new independent variable varies from 0 to 1. Then $F_\pm$ satisfies the differential equation

$$D_\pm(F_\pm) = 0,$$

$$D_\pm(F_\pm) \equiv p(p+1)\frac{d^2F_\pm}{dp^2}+(p+1)(1\mp 2iK')\frac{dF_\pm}{dp}+\lambda^2F_\pm$$

Applying the suggestion of Lanczos (1938) we now modify the differential equation by equating it to an error term proportional to Tshebysheff's polynomial $T_n(p)$ instead of to zero.

$$D_{\pm}(F_{\pm}) = \tau_{\pm} T_n(p) \qquad \dots \quad (5)$$

Let

$$T_n(p) = B_0 + B_1 p + B_2 p^2 + \dots + B_n p^n \qquad \dots \quad (6)$$

Now we put the finite series

$$\overline{F}_{\pm} = a_0 + a_1^{\pm} p + a_2^{\pm} p^2 + \dots + a_n^{\pm} p^n \qquad \dots \quad (7)$$

in the differential Eq. (5) and comparing the coefficients of the same power of p on both sides of the equation we get the recursion formulae

$$\tau_{\pm} B_r = a_r^{\pm}[\lambda^2 + r(r \mp 2iK')] + a_{r+1}^{\pm}[(r+1)(r+1 \mp 2iK'),$$

$$\tau_{\pm} B_n = a_n^{\pm}[\lambda^2 + n^2 \mp 2iK'n] \qquad \dots \quad (8)$$

These relations will determine the coefficients $a_r^{\pm}$ of the approximate solutions and the factor $\tau_{\pm}$ which estimates the error of the approximation in terms of a_0.

In the region $0 < r \leqslant R$ we write the solution as

$$u^i(x) = e^{i\gamma x} f_+ - e^{-i\gamma x} f_- \qquad \dots \quad (9)$$

where

$$\gamma = \sqrt{\lambda^2 + K'^2} \, ;$$

We make a transformation of the independent variable from x to $q = (e^x - 1)/(e^{x_0} - 1)$ such that the new independent variable varies from 0 to 1. Then $f_{\pm}$ satisfies the differential Eq.

$$q(q+1)\frac{d^2 f_{\pm}}{dq^2} + (q+1)(1 \pm 2i\gamma)\frac{df_{\pm}}{dq} - \lambda^2 f_{\pm} = 0 \qquad \dots \quad (10)$$

(neglecting terms involving $\beta \approx 10^{-6}$) we take

$$f_{\pm} = b_0 + b_1^{\pm} q + b_2^{\pm} q^2 + \dots + b_n^{\pm} q^n \qquad \dots \quad (11)$$

and get as before the recursion formulae to determine $b_r^{\pm}$ and $\tau_{\pm}'$ which estimates the error in terms of b_0.

$$\tau'_{\pm} B_r = b_r^{\pm}[-\lambda^2 + r(r \pm 2i\gamma)] + b_{r+1}^{\pm}[(r+1)(r+1 \pm 2i\gamma)]$$

$$\tau'_{\pm} B_n = b^{\pm}_n[-\lambda^2 + n(n \pm 2i\gamma)] \qquad \dots \quad (12)$$

From the continuity of the solution u and its derivative we obtain

$$\overline{\eta}_0 = e^{-2iK'x_0}\left[\frac{[(AG-CE)e^{2i\gamma x_0} + (CF-AH)}{(BG-DE)e^{2i\gamma x_0} + (DF-BH)}\right] \qquad \dots \quad (13)$$

where

$$A = \sum_{r=0}^{n} a_r^- \quad B = \sum_{r=0}^{n} a_r^+$$

$$C = -iK'A - \sum_{r=1}^{n} ra_r^-$$

$$D = iK'B - \sum_{r=1}^{n} ra_r^+$$

$$E = \sum_{r=0}^{n} b_r^+ ; \qquad F = \sum_{r=0}^{n} b_r^-$$

$$G = i\gamma E + \sum_{r=1} rb_r^+ ; \quad H = -i\gamma E + \sum_{r=1} rb_r^- ;$$

From equations (1) and (13) we obtain the expression for $\overline{\Gamma}_n/D$ and R'/R which are calculated with the following values of the parameters $V_0 = 52\text{MeV}$; $W = 3.12\text{ MeV}$; $R = (1.15A^{1/3} + 0.4)10^{-13}$ cm, $a = 0.57 \times 10^{-13}$ cm. The parameters are the same as taken by Feshbach *et al.* (1958) who obtained the neutron strength function by numerically solving the Schrödinger equation with the same potential. The curves of $\overline{\Gamma}_n{}^\circ/D$ (normalised to 1 ev) and R'/R are plotted against mass number

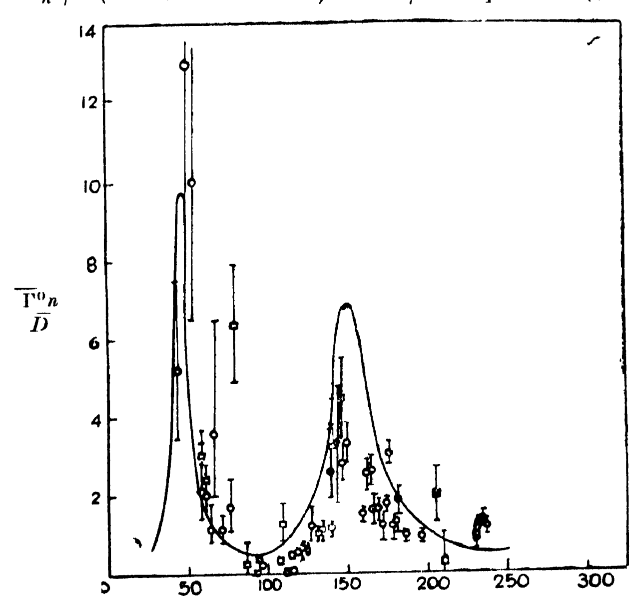

Fig. 1. Ratio $\overline{\Gamma}_n{}^0/D$ of neutron width to level spacing. Here $\overline{\Gamma}_n{}^0/D$ is normalized to 1 ev.

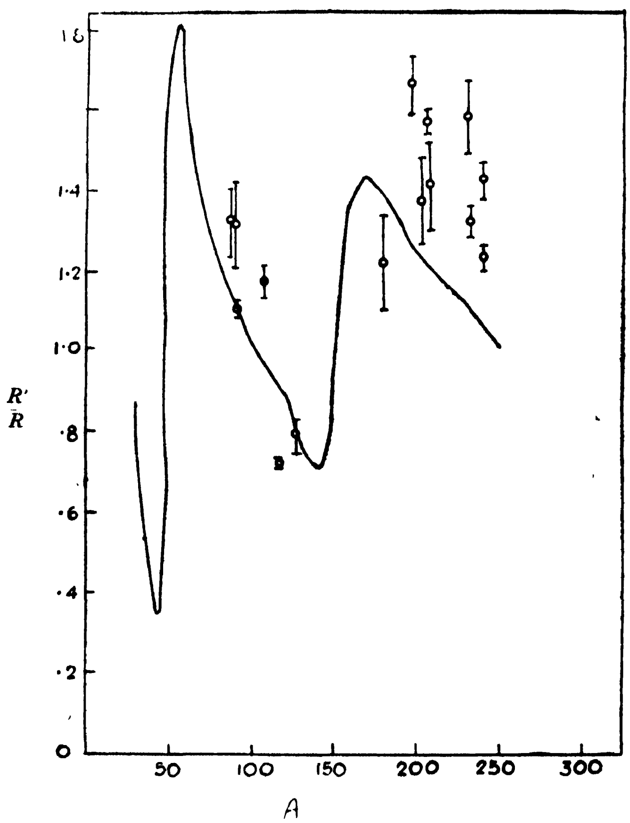

Fig. 2. Ratio of potential scattering length R′ to nuclear radius R.

in Fig. 1 and Fig. 2 and compared with the experimental results. In the experiment the minimum in $\Gamma_n{}^0/D$ between the two resonances is deeper than the theory predicts and the peak at $A = 155$ is much broader, lower and irregular than that of the calculated curve. A better fit of $\overline{\Gamma}_n{}^0/D$ at $A = 155$ has been obtained by Margolis *et al.* (1957) who has replaced the spherical square well by a spheroidal square well with the idea that the nuclei in this region are not spherically symmetrical. Similarly, in the R'/R curve closer agreement with the experimental results may be obtained for nuclei with $A = 200$ if the deformation of these nuclei is taken into account. In our curve of $\overline{\Gamma}_n{}^0/D$ the two maxima occur at $A = 48$ and 155 whereas in the calculation of Feshbach (1958) they occur at $A = 55$ and 155. Except for this shift of the peak point the two curves agree closely. The value of R'/R at about $A = 43$ is minimum in both the cases, but our value is numerically higher than that of Feshbach (1958); in all other regions the agreement between the two is good. Unfortunately there is no experimental result in the region $A = 43$ to indicate which one is superior to the other. In Fig. 3 we present for different elements the average total cross section (sum of elastic scattering and nuclear reactions) of neutrons of energy 500 eV in the unit of πR^2.

The series solution given here has the advantage that the number of terms to be taken in the polynomial is determined by the degree of accuracy one desires in

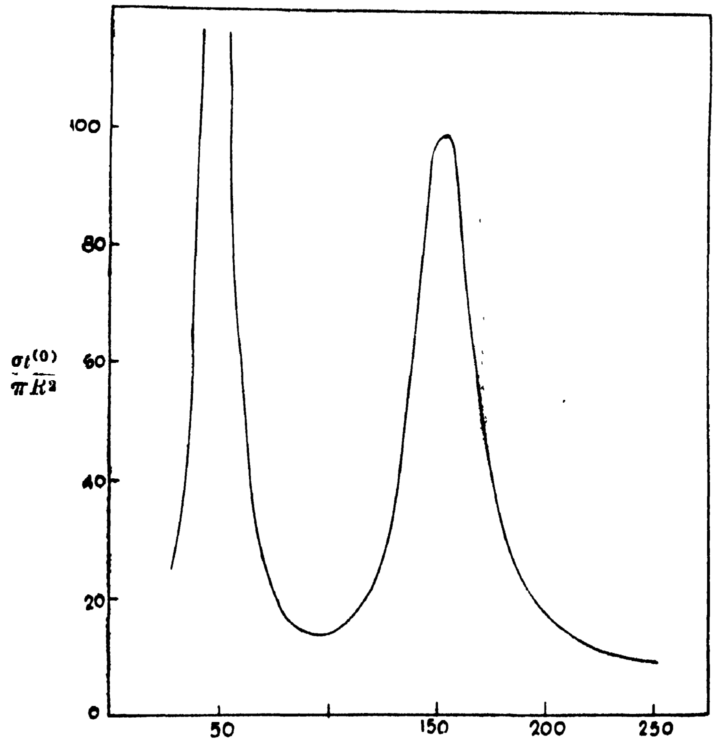

Fig. 3. Calculated total cross section of neutrons as a function of mass number.

the differential equation. For an accuracy of 1 in 10^6 in the differential equation it is sufficient to take a polynomial of the order of 8 whereas it is necessary to take several times this number of terms to achieve the same accuracy in Lawson's (1956) series.

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